Complex Analysis

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Abstract

A function $f : \mathbb{R}(x) \to \mathbb{R}(y)$ is real-analytic if it can be expanded in a power series,

$$y = f(x) = \sum_n a_n x^n.$$

A function

$$g : \mathbb{C}(z) \to \mathbb{C}(w)$$

is complex-analytic if it can be expanded in a power-series

$$w = g(z) = \sum_n c_n z^n.$$

Complex-analytic (also called holomorphic) functions can be characterized as solutions to the homogeneous Cauchy-Riemann equation $\frac{\partial g}{\partial \overline{z}} = 0$.

In complex analysis the inhomogeneous Cauchy-Riemann equation,

$$\frac{\partial g}{\partial \overline{z}} = u(z)$$

is an extremely important tool. It's main use is to produce holomorphic functions with powerful properties.
In this course we will explain the remarkable classical theory developed by Lars Hormander to handle this equation. We will focus on complex dimension one. This will make the proofs very simple and understandable, but will show all the ideas needed in the general higher dimensional case. The basic text is the book, Several complex variables by Hormander and notes from a course I gave in Beijing which added extra details to Hormanders book (which is a little brief at times).

[1] L. Hörmander, *An introduction to Complex Analysis in Several Complex Variables*, North-Holland.