COMPLEX ANALYSIS

Course Content

July 21-August 14, 2025-SMI Summer School Perugia

Professor Chiara de Fabritiis, Università Politecnica delle Marche, Ancona

Textbook: Theodore Gamelin, "Complex Analysis", Springer

Program: Chapter II-IX
Remark: The course is completely self-contained and presumes no previous knowledge of complex analysis.

Further indication for suggested reading before the course: Reviewing basic knowledge about one and two real variable calculus. We will use tools from calculus such as integration on curves, sequences and infinite sums and radius of convergence of power series.

More detailed (tentative) program

1. Elementary Properties of Analytic Functions
2. Complex Differentiation, Holomorphic Functions
3. Cauchy Integral Theorem (first version), Cauchy Integral Formula
4. Elementary Holomorphic Functions (exponential, sine, cosine, branches of the logarithm)
5. Liouville's, Gauss-Lucas and Morera's Theorem
6. Zeroes of Holomorphic Functions and Analytic Continuation
7. The Maximum Modulus Principle (weak form), the Open Mapping Theorem, the Maximum Modulus Principle (strong form), Weierstrass Theorem
8. Laurent Series, Isolated Singularities, Riemann Extension Theorem, Meromorphic Functions, Poles of a Meromorphic Function, Casorati-Weierstrass Theorem
9. Integration of Holomorphic Functions on Continuous Curves. Homotopy Version of Cauchy's Theorem. Existence of Primitives of Holomorphic Functions on Simply Connected Domains
10. Winding Number and the The Residue Theorem. The Argument Principle. Rouché's Theorem and the Inverse Function Theorems
11. The Schwarz' lemma and the Group of Automorphisms of the Unit Disk.
12. Z-transform, Inverse Z-transform, Difference Equations, Linear Digital Filters of Finite Order, Causal and non-Causal Filters.
13. Slice regular functions on domains of the quaternions $\mathbb{H}$.