FUNCTIONAL ANALYSIS

Scuola Matematica Interuniversitaria (SMI), Perugia

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Abstract:
In this course, we will develop functional and geometric methods to uncover special features exhibited by solutions of certain partial differential equations.
The fact is that, on the one hand, it would be desirable to "solve" all the partial differential equations that have some meaning for our existence, but no one knows how to do this.
On the other hand, you know, what doesn't kill you, adversity often breeds strength: the inability to find explicit solutions has spurred many beautiful minds to devise ingenious methods for understanding solutions without actually solving the equation!
This course will explore some of these classical discoveries, also establishing connections with mathematical structures that emerge in other disciplines.

Prerequisites:
Basic differential, multivariate, and integral calculus.
Basic topology.

References:
Serena Dipierro and Enrico Valdinoci.
Elliptic Partial Differential Equations from an Elementary Viewpoint. A Fresh Glance at the Classical Theory, World Scientific, 2024.
https://doi.org/10.1142/13776

Lawrence C. Evans.
Partial Differential Equations, American Mathematical Society, 2010.
https://bookstore.ams.org/gsm-19-r

Program:
What is the Laplacian and what is it good for?
Some classical partial differential equations.
The mean value formula.
The Laplace-Beltrami operator.
The Kelvin Transform
The fundamental solution
Maximum Principles
The Green Function
The Poisson Kernel
Analyticity of harmonic functions
The Harnack Inequality
The Hopf Lemma
Cauchy's Estimates
The Weak Harnack Inequality

The Boundary Harnack Inequality
Liouville's Theorem
Harmonic polynomials and spherical harmonics
Potential theory and Schauder estimates for the Laplace operator
Pointwise Hölder spaces
Schauder estimates
Sobolev spaces
Introduction to the regularity theory in Lebesgue spaces
Applications