SMI Summer School 2024
Differential Geometry

Course Program

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Short description

This course explores the mathematical terrain of Differential and Riemannian Geometry, trying to offer a comprehensive understanding of fundamental concepts and advanced theorems essential in modern geometric analysis. If time permits, in the advanced segment notions of Morse Theory, and or Mathematical Relativity and causality in spacetimes will be discussed.

Tentative program

- **Basics of Differential Geometry**
  - Topological and differentiable manifolds. Manifolds with boundary.
  - Tangent and cotangent bundle, vector fields and 1-forms.
  - Partitions of unity. Whitney Embedding Theorem.
  - Local form of immersions and submersions.
  - Distributions, integrability and Frobenius Theorem.
  - Lie groups and Lie algebras.
  - Group actions on manifolds.
- **Riemannian Geometry**
  - Riemannian metrics
  - Levi–Civita connection and curvature.
  - Distance, completeness, geodesics. Convex neighborhoods.
  - Hopf–Rinow Theorem.

  - Variational calculus. First and second variational formulas.
  - Jacobi fields, conjugate points, the Morse Index Theorem.
  - Rauch Comparison Theorem. Hadamard Theorem.
  - Submanifolds, second fundamental form, focal points
  - Isometric immersions:fundamental equations.

- **Additional topics (if time permits)**
  - The Sphere Theorem
  - Notions of Morse Theory.
  - Basics of Lorentzian Geometry and Mathematical Relativity.

Suggested references

1. J. K. Beem, P. Ehrlich, K. Easley, *Global Lorentzian Geometry*, 2nd Edition, Chapman & Hall, 1996.

2. S. Kobayashi, K. Nomizu, Katsumi, *Foundations of differential geometry*, Vol I. John Wiley & Sons, 1963.

3. S. Kobayashi, K. Nomizu, Katsumi,*Foundations of differential geometry*, Vol. II., John Wiley & Sons, 1969 1969.

4. M. P. do Carmo, *Riemannian Geometry*, Birkhauser, 1992.

5. F. Mercuri, P. Piccione, D. Tausk, *Classical and Modern Morse Theory with Applications*, IMPA, 2001.